The Model of Population Dynamics of Root Hemiparasitic Plants along a Productivity Gradient

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Outline

Mathematical models

- Description of some system in mathematic (eg. with differential equations)
- Model is always constrained
- Focus on the most important things
- Many simplifications
- Goal is formal representation of major parts of the system

Model of D. Smith [1]

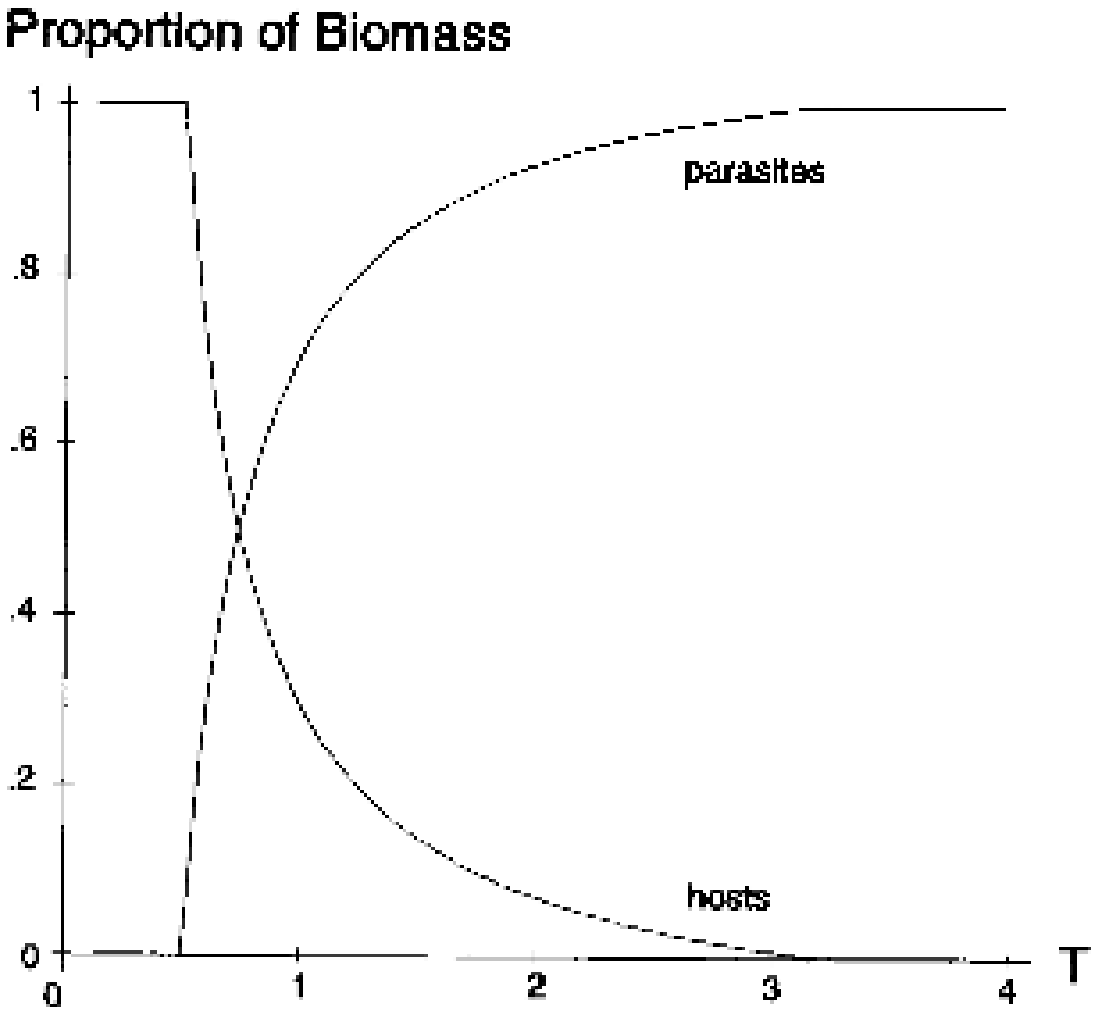
- Autonomous system

$$\begin{aligned}
\frac{dp}{dt} &= p * (trophic_p(z_p) - mortality_p) \\
\frac{dh}{dt} &= h * (trophic_h(z_h) - mortality_h) \\
z_p &= r + \gamma * h \\
z_h &= r - IMPACT * p \\
r &= PRODUCTIVITY - p - h
\end{aligned}$$

- p, h - biomass of parasite, host
- z_p, z_h - available resources for parasites, hosts
- $\gamma, IMPACT, PRODUCTIVITY$ are constants

[1] Smith, D. 2000. The population dynamics and community ecology of root hemiparasitic plants.American Naturalist 155:13:23.

Model of D. Smith - result



Model of D. Smith - problems

- Constant mortality
- Increasing PRODUCTIVITY do not increase biomass of hemiparasitic plants
  - At high productivities is more important above ground competition
  - $z_p = r + \gamma * h \Rightarrow$ hemiparasitic plants could grow well without host only on resources from soil, but for many hemiparasitic plants is hemiparasitic strategy obligatory

Our model - overview

- Autonomous system

$$\begin{aligned} \frac{dp}{dt} &= p * (trophic_p(z_p) - mortality_p(p)) \\ \frac{dh}{dt} &= h * (trophic_h(z_h) - mortality_h(h)) \end{aligned}$$

- $p, h > 0$ - biomass of parasites, hosts
- t denotes time
- z_p, z_h - available resources for parasites, hosts
- $trophic_{p,h}$ - trophic functions of parasites, hosts

Mortality

- Definition

$$mortality_{p,h}(species) = species * CMORTALITY_{p,h}$$

- $CMORTALITY_{p,h}$ denotes part of species biomass that returns to the soil

$$0 < CMORTALITY_{p,h} < 1$$

$$\begin{aligned} CMORTALITY_h &\ll CMORTALITY_p \\ &\Rightarrow \\ mortality_h(s) &\ll mortality_p(s),\ \forall s > 0 \end{aligned}$$

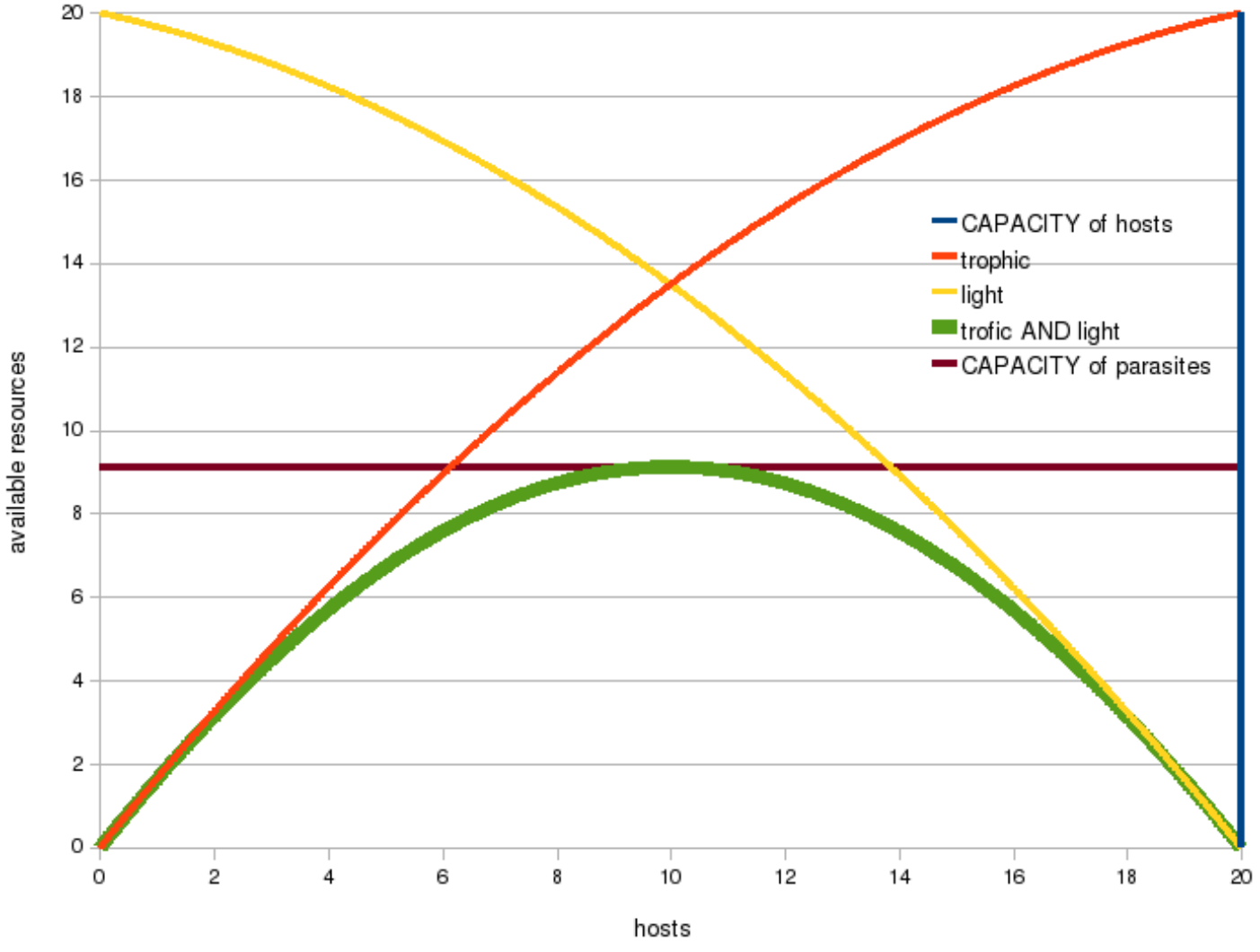
Host trophic function

- $trophic_h(z_h)$

$$\begin{aligned} z_h &= r - IMPACT * p \\ r &= PRODUCTIVITY - p - h \end{aligned}$$

- $dtrophic_h(z_h)/dz_p > 0 \Rightarrow trophic_h(z_h) \nearrow$
- z_h - available resources for hosts
- $0 < PRODUCTIVITY$ (richness) of the system
- $0 \leq r \leq PRODUCTIVITY$ - available resources in the soil
- $0 < IMPACT$ - impact of parasites on hosts

Parasite trophic function



Parasite trophic function - formal

- $trophic_p(z_p)$, z_p is parabole ($\nearrow\searrow$)

$$z_p = \frac{h - CAPACITY_h/2}{2 * CPARABOLE} + CAPACITY_p$$
$$CPARABOLE = -\frac{2 * CAPACITY_p}{(CAPACITY_h/2)^2}$$

- z_p combination of resources from host ($\nearrow$) and light availability ($\searrow$)
- $CAPACITY_h < PRODUCTIVITY$ - capacity of host
- $CAPACITY_p < PRODUCTIVITY$ - capacity of parasite

$$CAPACITY_p \ll CAPACITY_h$$

Low PRODUCTIVITY and *Rhinanthus minor*

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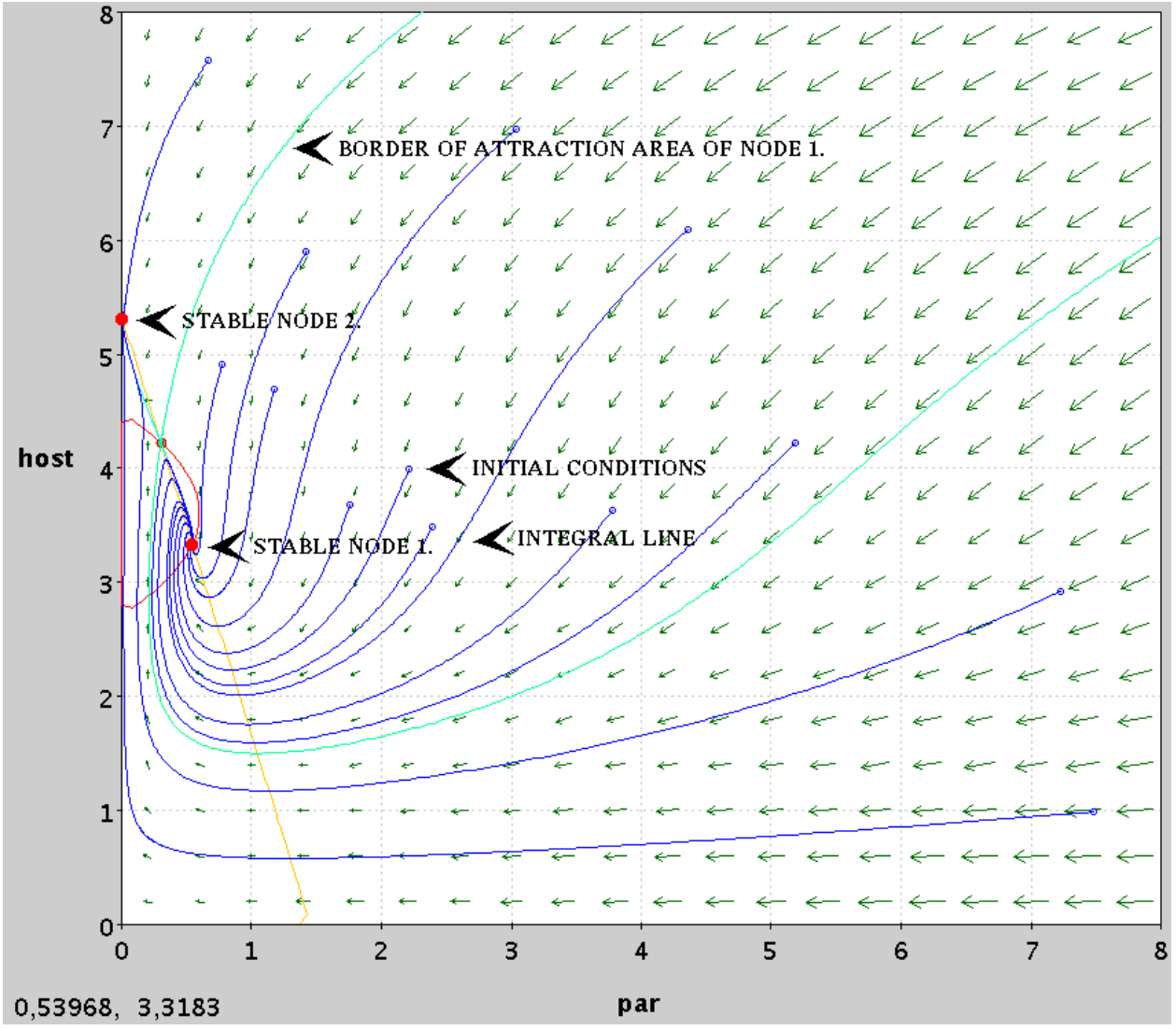
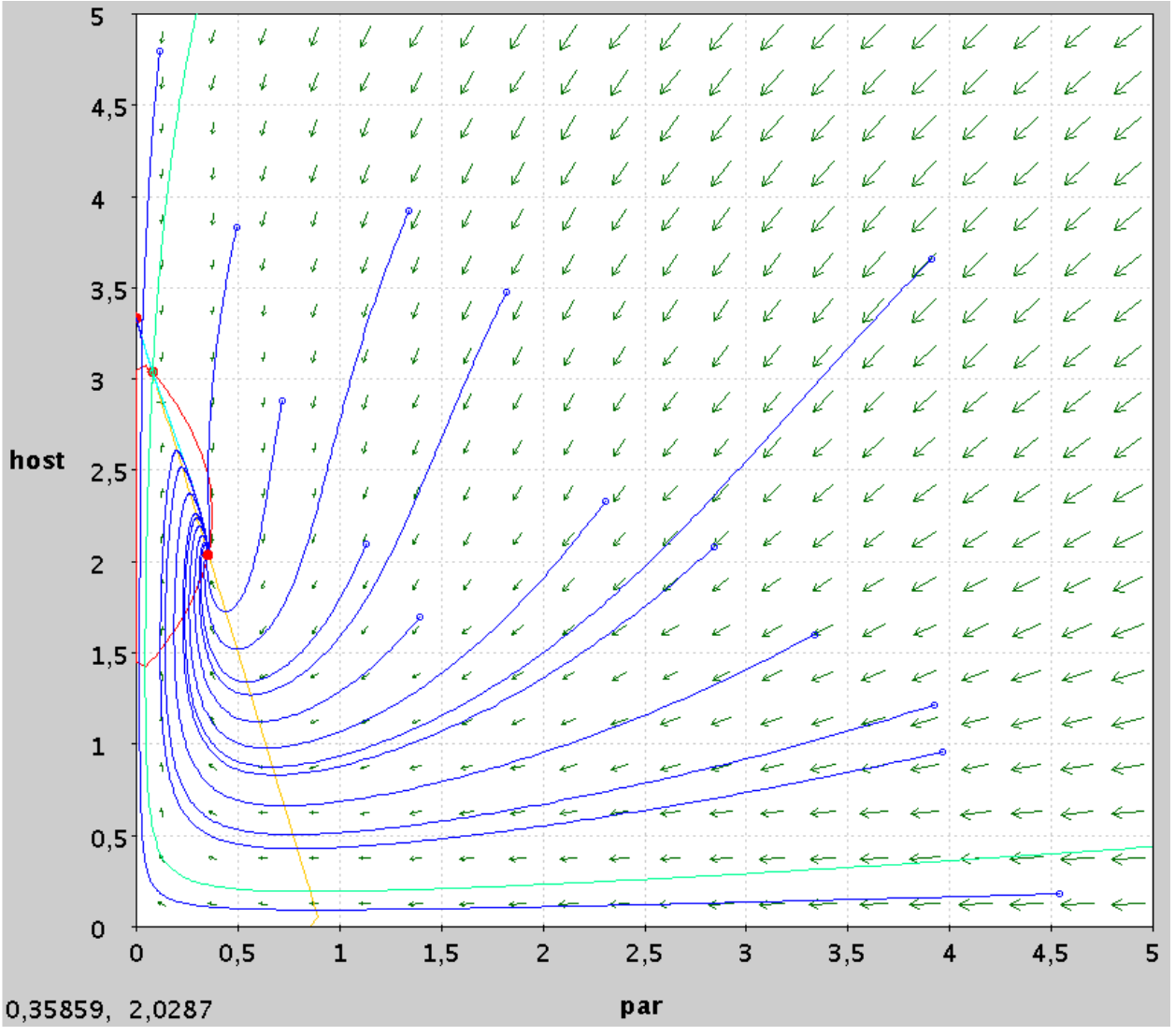
What is simplified

- Fixed productivity of system
- Symetric trophic function
- No disturbance
- No variability and distribution of individuals
- No distribution of resources from soil

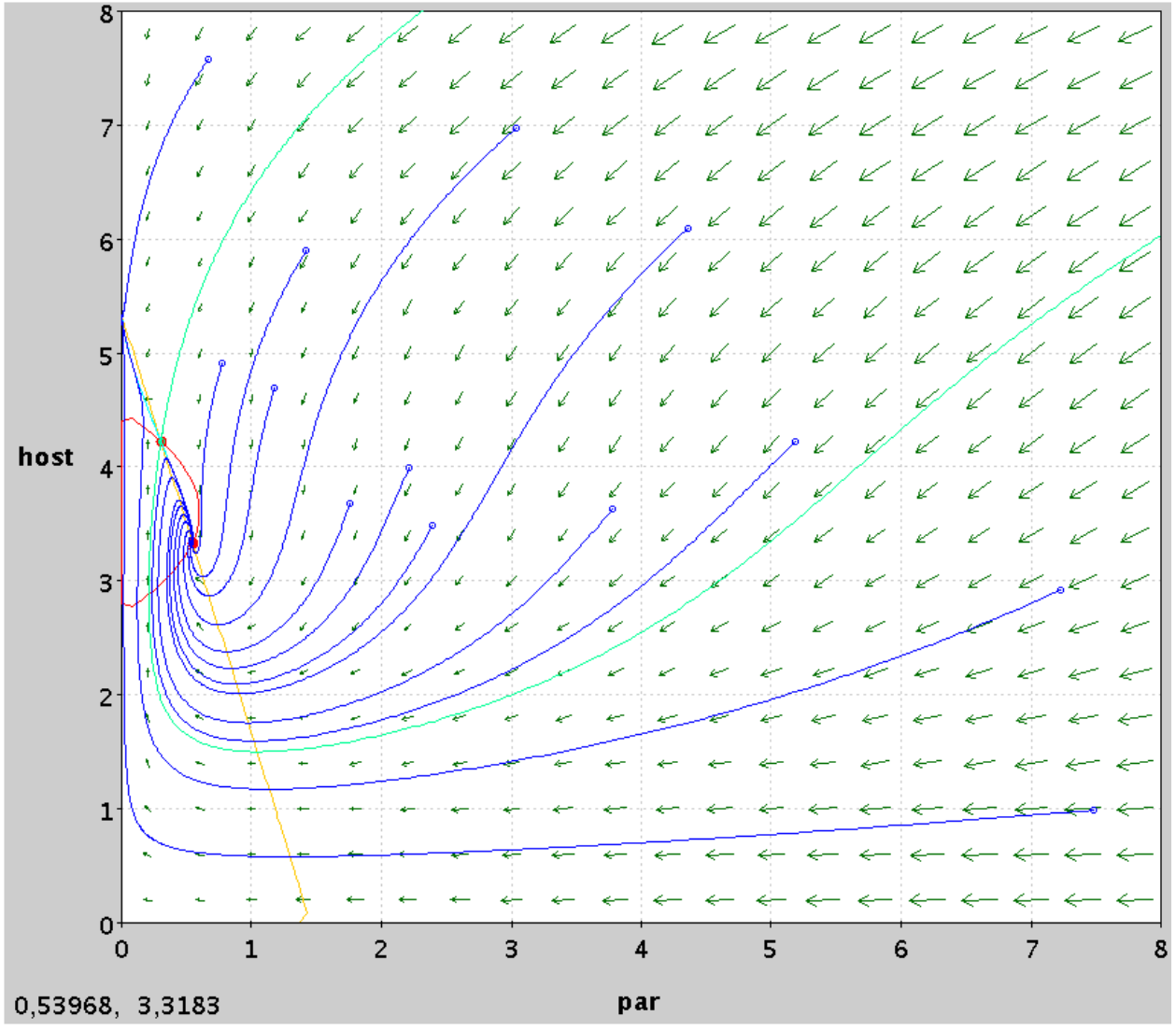
Example of phase plane



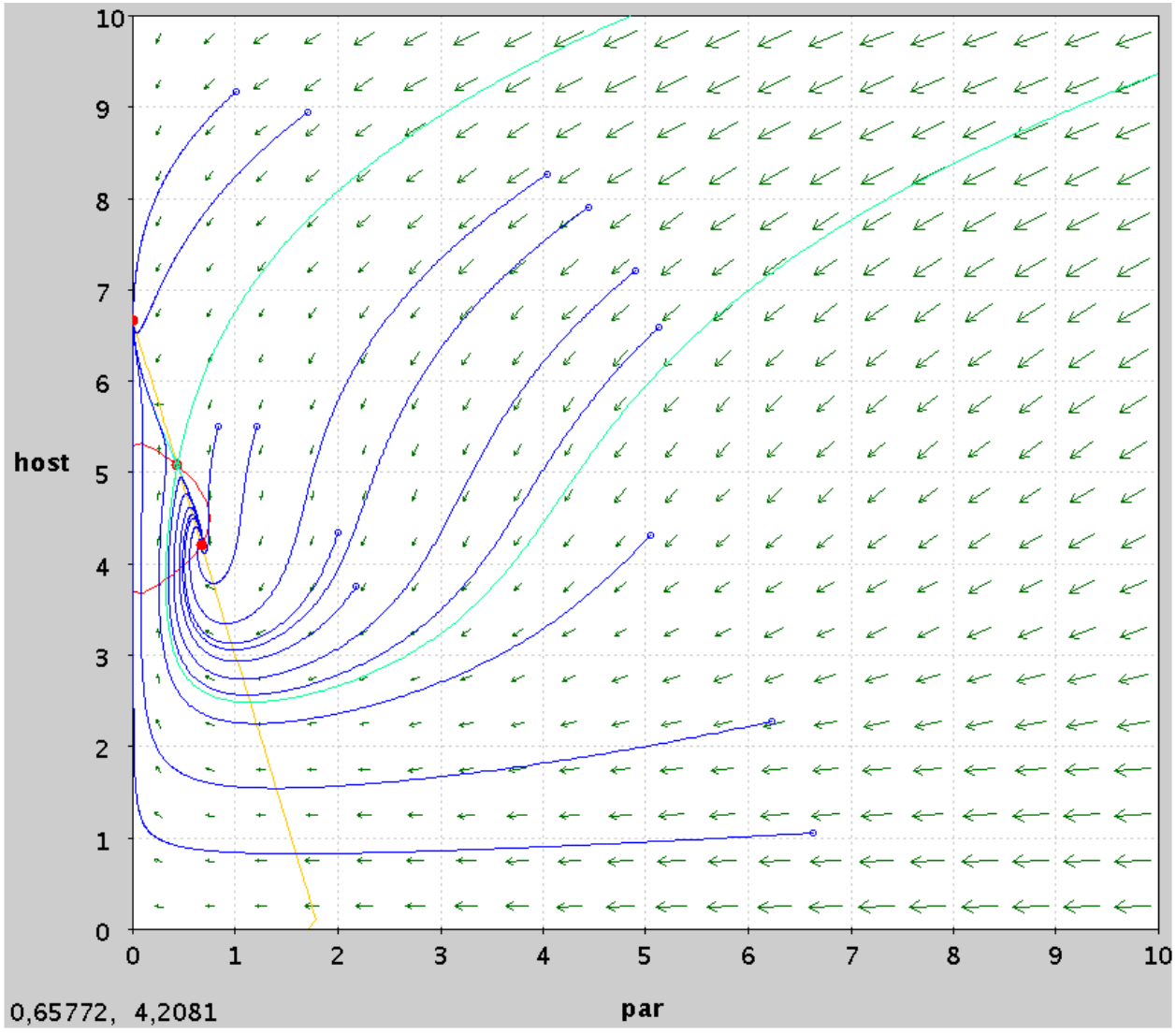
Phase plane, PRODUCTIVITY=5



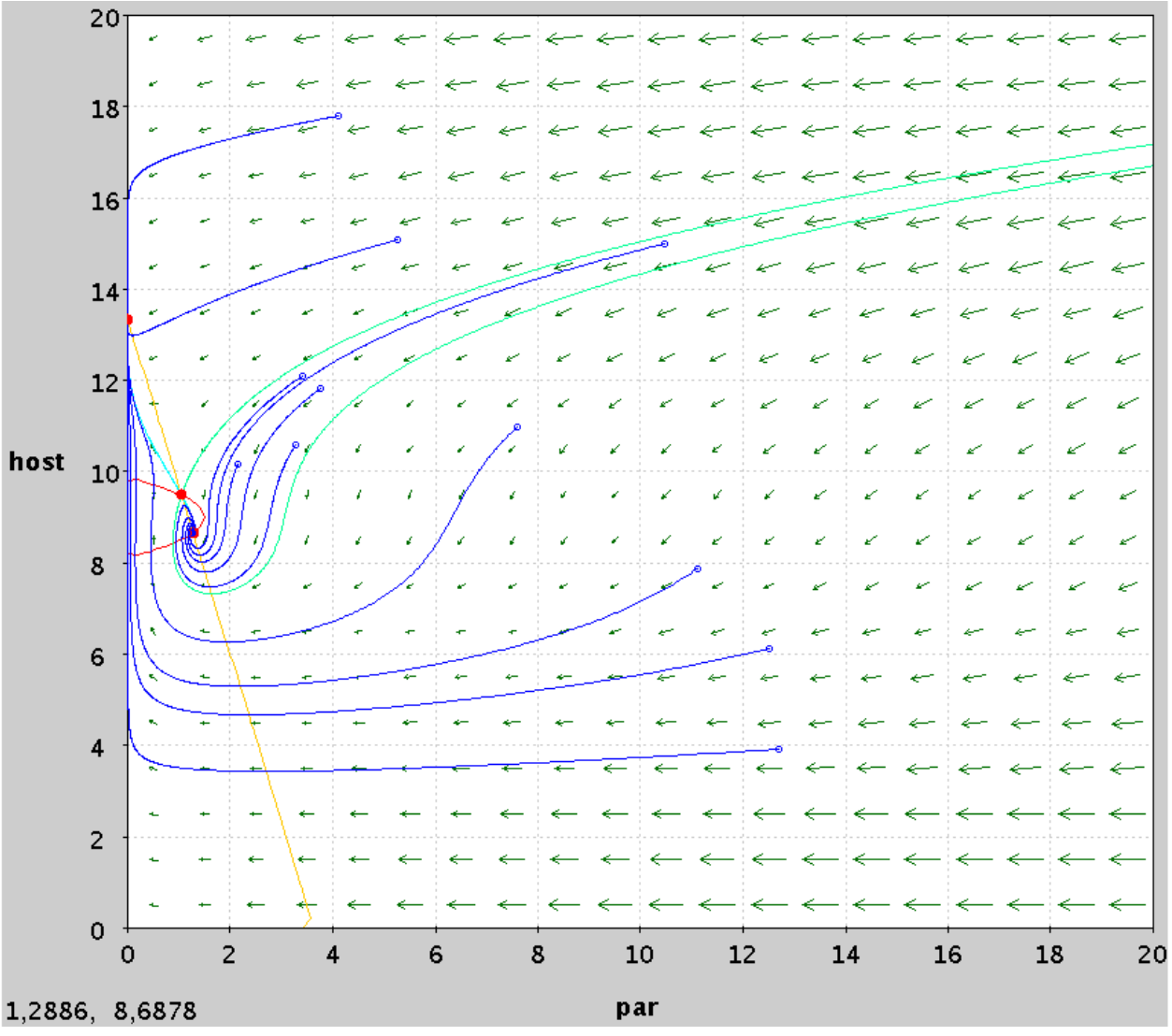
Phase plane,PRODUCTIVITY=8



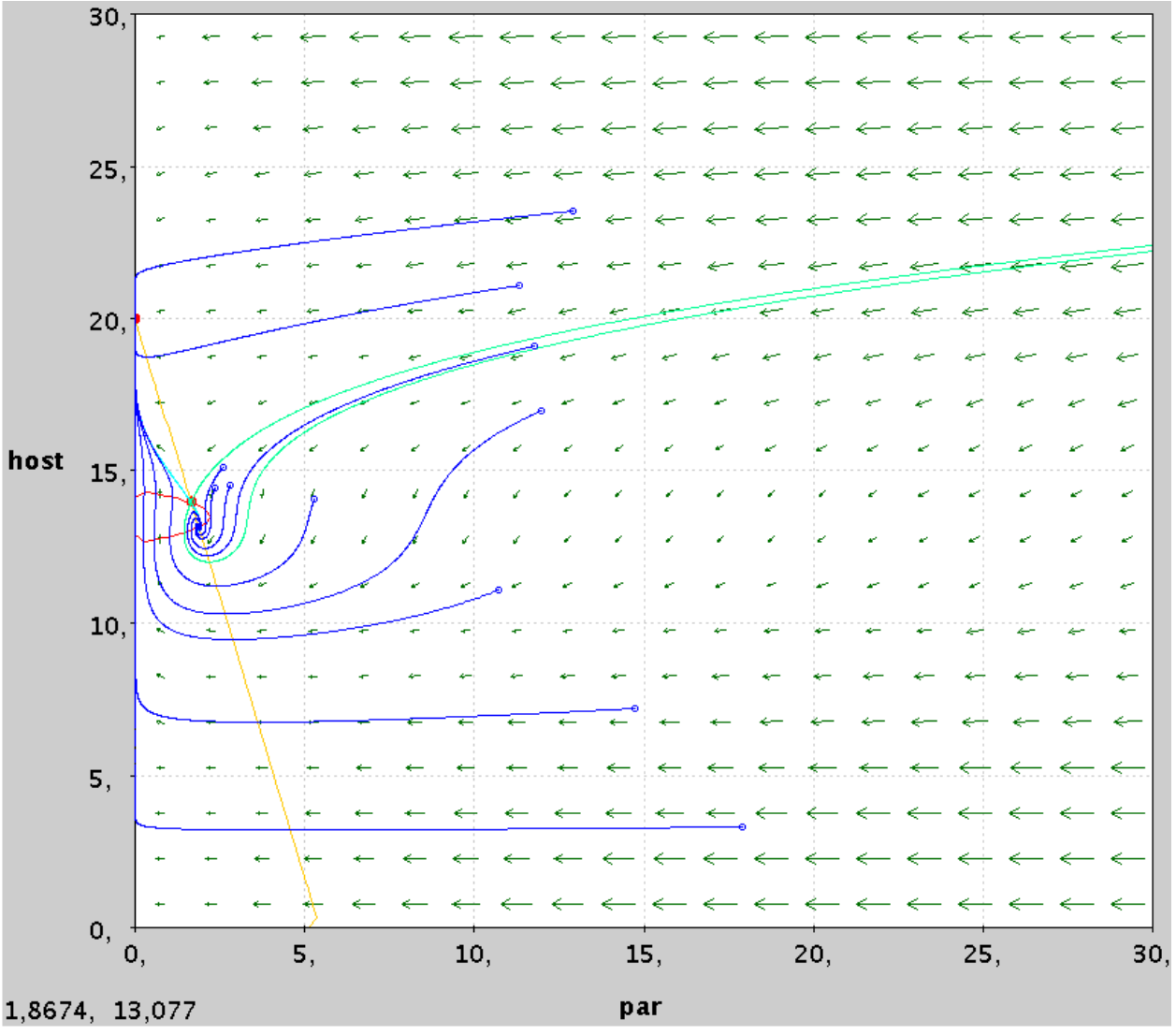
Phase plane,PRODUCTIVITY=10



Phase plane,PRODUCTIVITY=20



Phase plane,PRODUCTIVITY=30

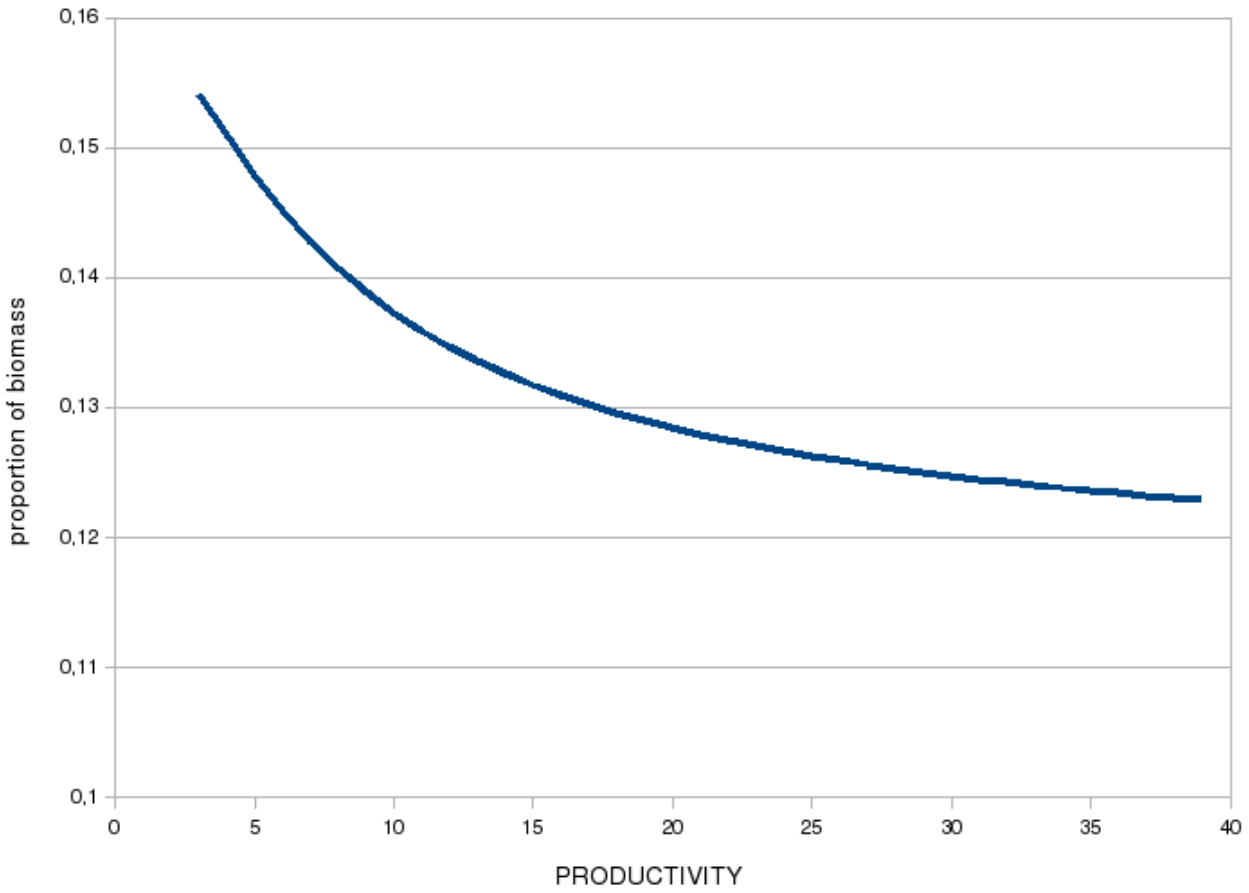


Phase planes results

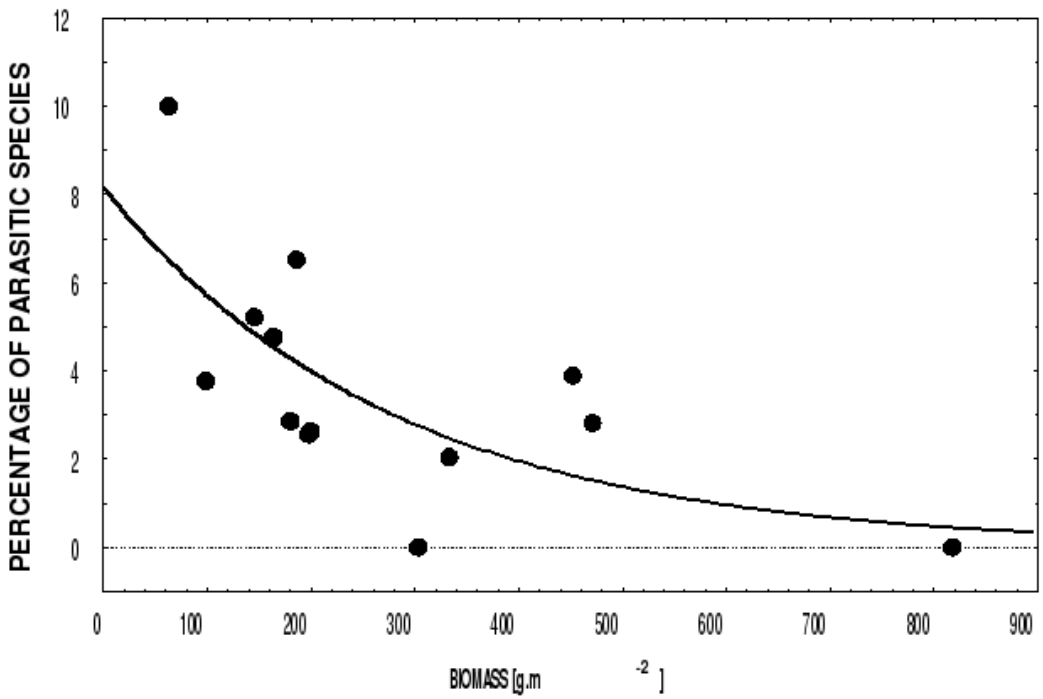
Attraction area is reducing along productivity gradient

- dependence on initial conditions, that lead to coexistence, is growing with productivity $\Rightarrow$ chance to coexistence is decreasing along productivity gradient
- increasing productivity leads coexisting system to be less tolerant to disturbance

Proportion of biomass of parasitic plants at stable point along a productivity gradient



Relationship between the proportion of hemiparasitic plants and biomass of vascular plants [2]



[2] Petru, M.,and J. Lepš made data analyse on Hadač, E. 1969. Die Pflazengesellshaften des Tales "Dolina Siedmich prameňov" in der Balear Tatra. [Plant communities of the valley "Dolina Siedmich prameňov" *in* the Belianske Tatry Mts.] Vydavatelstvo Slovenkej Akademie Vied, Bratislava.

Low PRODUCTIVITY and *Rhinanthus minor*

Discussion

- We add 2 properties (dynamic mortality, aboveground competition) of original system that model of D. Smith has not
- Our model matches field observations, Petru, M. and J. Lepš result, Matthies result[3] that with increasing productivity is abundance of hemiparasitic plants decreasing

[3] Matthies, D. 1995. Parasitic and competitive interactions between the hemiparasites *Rhinanthus serotinus* and *Odontites rubra* and their host *Medicago sativa*. Journal of Ecology 83:245-251.

Future work

- Focus on parasite trophic function and its combination with light availability function
- To cover and quantify other major parts of the system of hemiparasitic plants and their hosts

Questions?